

# Geometry: Measuring Three-Dimensional Figures



## “What do paint cans have to do with math?”

Paint cans come in many different sizes, but they are all shaped like cylinders. To find the volume  $V$  of a paint can, you can use the formula  $V = \pi r^2 h$ , where  $r$  is the radius of the lid and  $h$  is the height of the can. A different formula can be used to find the surface area of a paint can.

You will solve problems involving volumes and surface areas of cylinders in Lessons 12-3 and 12-5.

# GETTING STARTED

## ► Diagnose Readiness

Take this quiz to see if you are ready to begin Chapter 12. Refer to the lesson or page number in parentheses for review.

### Vocabulary Review

Choose the correct term to complete each sentence.

- The distance around a circle is called (perimeter, circumference). (Lesson 6-9)
- The expression  $\pi r^2$  is used to find the (circumference, area) of a circle. (Lesson 11-6)

### Prerequisite Skills

Estimate each product. (Page 558)

3.  $9 \cdot 10.4$     4.  $6.25 \cdot 3.8$     5.  $7.03 \cdot 5.3$

Evaluate each expression. Round to the nearest tenth if necessary. (Pages 559, 560)

6.  $14.45 + 9.62$     7.  $8.3 \cdot 6.4$   
 8.  $36 \times 5.2$     9.  $26.45 - 7.918$

Simplify. (Lesson 1-3)

10.  $2 \cdot 6 + 4 \cdot 1 \cdot 3$     11.  $1.2 \cdot 4 \cdot 4 + 3 \cdot 1.5$   
 12.  $8 + 1 + 9 \cdot 2 + 5$     13.  $7 + 3 \cdot 5 + 2 \cdot 6$

Multiply. (Lesson 6-4)

14.  $4\frac{1}{2} \cdot 6$     15.  $1\frac{1}{4} \cdot 5\frac{3}{5}$     16.  $\frac{2}{7} \cdot 3\frac{3}{4}$

Find the area of each circle. Round to the nearest tenth. (Lesson 11-6)

17. diameter = 33 cm    18. radius = 3.8 yd  
 19. radius = 6 ft    20. diameter = 18 m



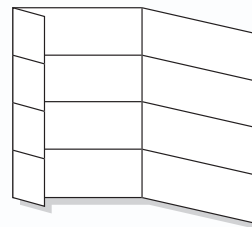
### Surface Area and Volume

Make this Foldable to help you organize information about solids. Begin with a piece of 11" by 17" paper.

**STEP 1** **Fold** Fold the paper in fourths lengthwise.

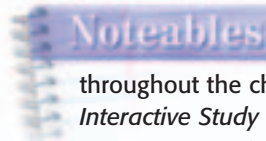


**STEP 2** **Open and Fold** Fold a 2" tab along the short side. Then fold the rest in half.



**STEP 3** **Label** Draw lines along folds and label as shown.

Ch. 12	Rectangular Prisms	Cylinders
Draw Examples		
Find Volume		
Find Surface Area		



### Chapter Notes

Each time you find this logo throughout the chapter, use your *Noteables™*: *Interactive Study Notebook with Foldables™* or your own notebook to take notes. Begin your chapter notes with this Foldable activity.



**Readiness** To prepare yourself for this chapter with another quiz, visit [msmath2.net/chapter\\_readiness](http://msmath2.net/chapter_readiness)

#### What You'll LEARN

Build three-dimensional figures given the top, side, and front views.

#### Materials

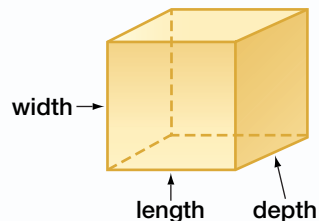
- centimeter cubes

#### STUDY TIP

**Plane** A *plane* is a flat surface that extends in all directions. Each face of a cube represents a different plane.

### Building Three-Dimensional Figures

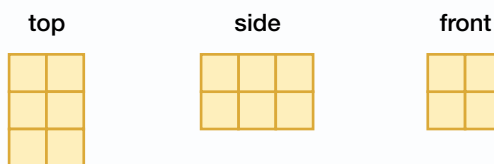
Cubes are examples of three-dimensional figures because they have length, width, and depth. In this lab, you will use centimeter cubes to build other three-dimensional figures.



#### ACTIVITY

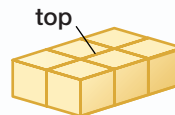
Work with a partner.

The top view, side view, and front view of a three-dimensional figure are shown below. Use centimeter cubes to build the figure.



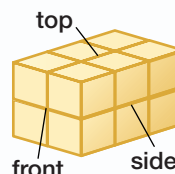
#### STEP 1

Use the top view to build the base of the figure. It is a 3-by-2 rectangle.



#### STEP 2

Use the side view to complete the figure. It is a 2-by-3 rectangle.

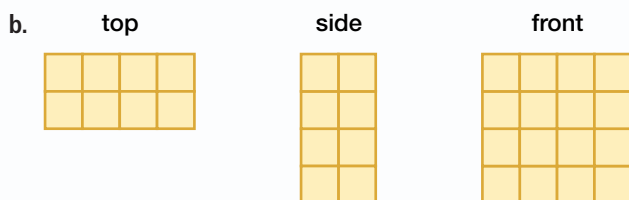
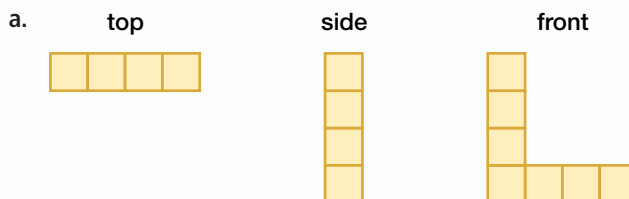


#### STEP 3

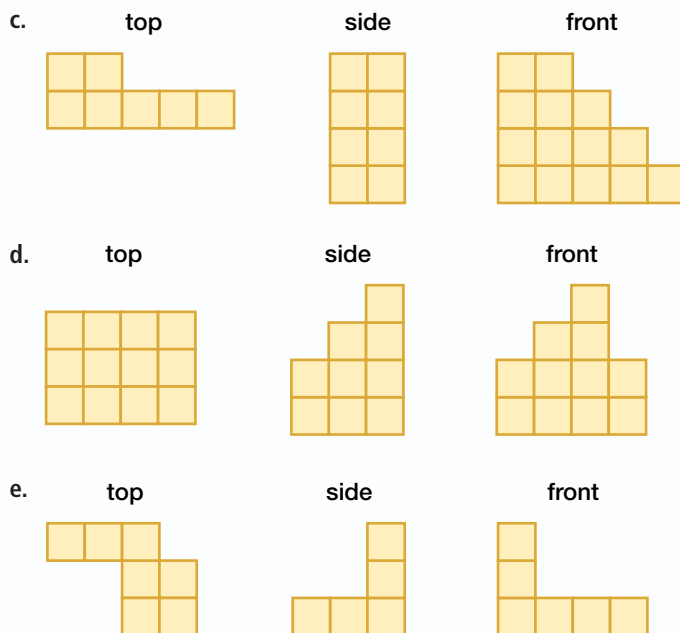
Use the front view to check the figure. It is a 2-by-2 square. So, the model is correct.

#### Your Turn

The top view, side view, and front view of each three-dimensional figure are shown. Use centimeter cubes to build the figure. Then make a sketch of the figure.



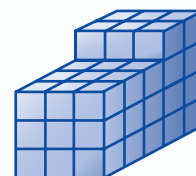
The top view, side view, and front view of each three-dimensional figure are shown. Use centimeter cubes to build the figure. Then make a sketch of the figure.



## Writing Math

*Work with a partner.*

- Build** a model with cubes and draw the top, side, and front views. Give the drawing of the views to your partner and have him or her build the figure with cubes. Trade roles with your partner and repeat making the drawing and building the figure.
- Explain** how you began building the figures.
- Determine** whether there is more than one way to build each model. Explain your reasoning.
- The figure at the right represents a building with a section that is 15 stories tall and another section that is 20 stories tall. Which view would you use to show the difference in height of each section?
- Build** two different models that would look the same from two views, but not the third view. Draw a top view, side view, and front view of each model.
- Describe** a real-life situation where it might be necessary to draw a top, side, and front view of a three-dimensional figure.



## Drawing Three-Dimensional Figures

### What You'll LEARN

Draw a three-dimensional figure given the top, side, and front views.

### NEW Vocabulary

solid

**WHEN** am I ever going to use this?

**COMICS** For Exercises 1 and 2, refer to the comic below.

#### SHOE

by Jeff MacNelly



1. Which view of the Washington Monument is shown in the comic?
2. Find a photograph of the Washington Monument and draw a side view.

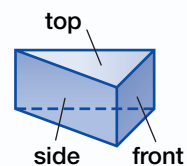
### STUDY TIP

**Plane Figures** In geometry, three-dimensional figures are *solids* and two-dimensional figures such as triangles, circles, and squares are *plane figures*.

A **solid** is a three-dimensional figure because it has length, width, and depth. You can draw different views of solids.

### EXAMPLE Draw Different Views of a Solid

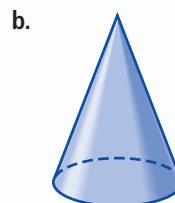
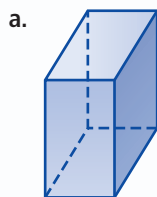
- 1 Draw a top, a side, and a front view of the figure at the right.



The top view is a triangle.  
The side and front views are rectangles.



- 2 **Your Turn** Draw a top, a side, and a front view of each solid.



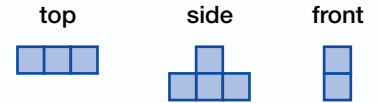
## STUDY TIP

**Paper** Use isometric dot paper for the drawings in this lesson, as shown at the right. This is different from rectangular dot paper.

### EXAMPLE

### Draw a Three-Dimensional Figure

- 1 Draw the solid using the top, side, and front views shown at the right.



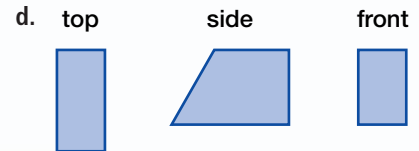
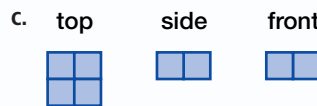
**Step 1** Use the top view to draw the base of the figure, a 1-by-3 rectangle.

**Step 2** Add edges to make the base a solid figure.

**Step 3** Use the side and front views to complete the figure.

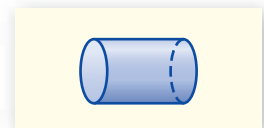
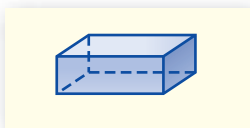


- 2 **Your Turn** Draw each solid using the top, side, and front views shown. Use isometric dot paper.



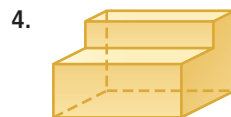
## Skill and Concept Check

- OPEN ENDED** Draw the top, side, and front view of a solid in your home.
- Which One Doesn't Belong?** Identify the figure that does not have the same characteristic as the other three. Explain your reasoning.

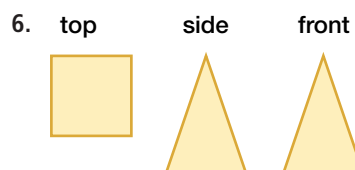
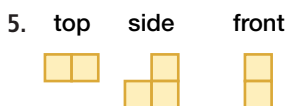


## GUIDED PRACTICE

Draw a top, a side, and a front view of each solid.



Draw each solid using the top, side, and front views shown. Use isometric dot paper.



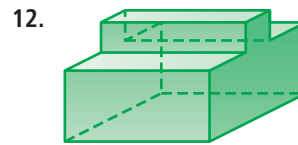
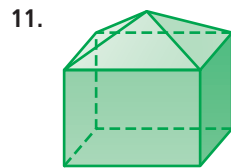
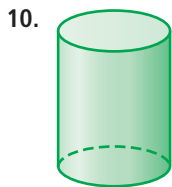
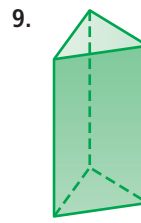
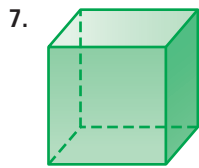
# Practice and Applications

**HOMWORK HELP**

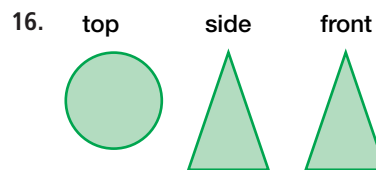
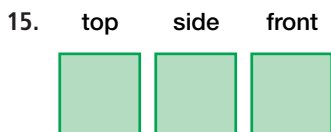
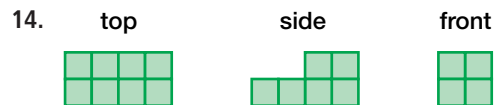
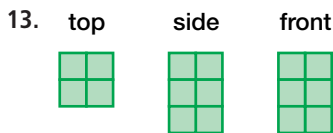
For Exercises	See Examples
7–12, 18	1
13–17, 19, 20	2

**Extra Practice**  
See pages 593, 607.

Draw a top, a side, and a front view of each solid.



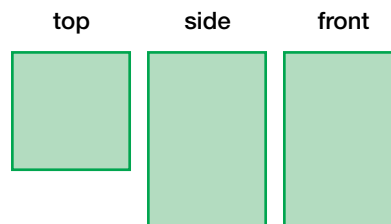
Draw each solid using the top, side, and front views shown. Use isometric dot paper.



**HISTORY** For Exercises 17–20, use the information below.

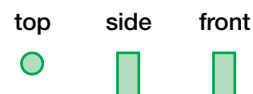
The Lighthouse of Alexandria was the last of the seven Wonders of the Ancient World that disappeared. It contained three different-shaped levels.

17. The bottom level is the largest. Make a drawing of this level using the top, side, and front views shown at the right.



18. The middle level is a prism with a base that is a regular octagon. If the height and width of this level is approximately one third the height and width of the bottom level, draw the top, side, and front views.

19. The top level has the views shown at the right. What kind of solid is this?



20. Make a sketch of the Lighthouse of Alexandria that shows all three levels.



**Data Update** How does your drawing compare to the actual Lighthouse of Alexandria? Visit [msmath2.net/data\\_update](http://msmath2.net/data_update) to learn more.

21. **RESEARCH** Use the Internet or another source to find a photograph of the only Wonder of the Ancient World existing today, the Great Pyramid of Giza. Draw a top view, a side view, and a front view of the pyramid.
22. **ARCHITECTURE** When a building is being designed, an architect provides a set of elevation drawings. These drawings show how the building appears from each side. Draw a set of elevation drawings for your home or school.
23. **CRITICAL THINKING** Draw a three-dimensional figure in which the front and top views each have a line of symmetry but the side view does not. (*Hint: Refer to Lesson 10-9 to review lines of symmetry.*)

## Spiral Review with Standardized Test Practice

24. **MULTIPLE CHOICE** Which is the top view of the cylinder at the right?

- (A) triangle                      (B) rectangle  
(C) square                        (D) circle



25. **MULTIPLE CHOICE** Which three-dimensional figure has the top, side, and front views shown at the right?

(F)



(G)



top

side

front



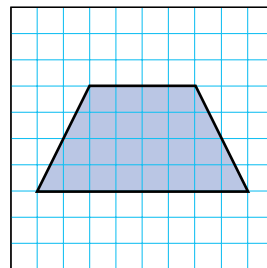
(H)



(I)



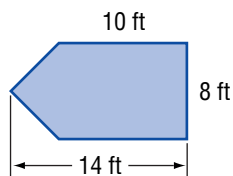
26. Find the probability that a randomly dropped counter will fall in the shaded region. Write as a percent. (*Lesson 11-8*)



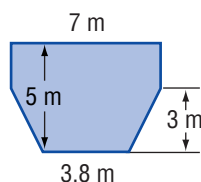
Find the area of each figure. Round to the nearest tenth if necessary.

(*Lesson 11-7*)

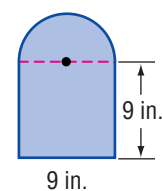
27.



28.



29.



### GETTING READY FOR THE NEXT LESSON

**PREREQUISITE SKILL** Multiply. (*Lesson 6-4*)

30.  $7\frac{1}{2} \cdot 6$

31.  $8 \cdot 2\frac{3}{4}$

32.  $\frac{5}{6} \cdot 1\frac{4}{5}$

33.  $10\frac{1}{5} \cdot 6\frac{2}{3}$



# 12-1b

## Problem-Solving Strategy

A Follow-Up of Lesson 12-1

### Make a Model

#### What You'll LEARN

Solve problems by making a model.

I'm going to help my mom make 3-inch soft alphabet blocks for the children at her daycare center. About how much fabric do we need for one cube if there is a  $\frac{1}{2}$ -inch seam on each side?

We could **make a model** to find out.

#### Explore

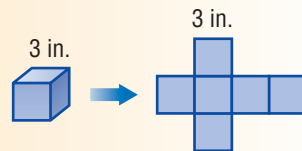
We know that each cube is 3 inches long with  $\frac{1}{2}$ -inch seams.

#### Plan

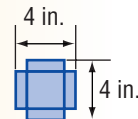
We can make a cardboard model of a cube with sides 3 inches long. We could then cut the model into six squares and add  $\frac{1}{2}$ -inch paper extensions to each side as seams.

#### Solve

Make the cardboard model and unfold the cube.



Cut the cube into six squares and tape  $\frac{1}{2}$ -inch seams on each side. Now each pattern piece is 4 inches by 4 inches or *about* 16 square inches.



$$16 \text{ in}^2 \times 6 \text{ squares} = 96 \text{ in}^2$$

So, each cube needs about 96 square inches of fabric.

#### Examine

If the 4-inch-by-4-inch pattern pieces were laid as two columns and three rows, it would be 8 inches wide and 12 inches long or 96 square inches.

### Analyze the Strategy

1. **Explain** when making a model is a better strategy than drawing a picture.
2. **Explain** why you think the students started with the three-dimensional model to make their pattern.
3. **Write** a problem that can be solved by making a model. Then solve the problem.

## Apply the Strategy

**Solve. Make a model.**

4. **ART** Dominic is creating a layout of his bedroom for art class. The room measures 15 foot by 12 feet. If he uses a scale of  $1 \text{ foot} = \frac{3}{4} \text{ inch}$ , what are the dimensions of his bedroom on the model?
5. **BICYCLES** Eight customers lined up outside The Bike Shop with either a bicycle or a tricycle that needed repair. When the owner looked out the window, she counted 21 wheels outside the shop. How many tricycles and bicycles were there?

## Mixed Problem Solving

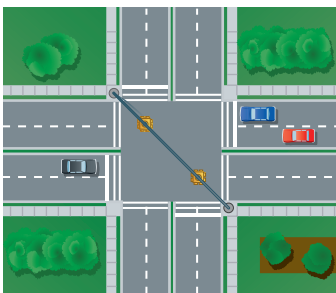
**Solve. Use any strategy.**

6. **COMMUNITY SERVICE** There are four drop-off centers for the community food drive. Their total collections are shown in the table.

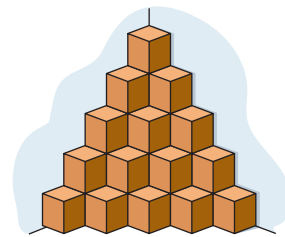
Center	Number of Cans
A	3,298
B	2,629
C	4,429
D	2,892

A newsletter reported that over 13,000 cans of food were collected. Is this estimate reasonable? Explain.

7. **SWIMMING** Yeti can swim one 20-meter lap in 1.25 minutes. How long will it take her to swim 100 meters at the same rate?
8. **TRAFFIC** At the four-way intersection shown below, the traffic lights change every 90 seconds. About 8 cars in one lane travel through the light in this amount of time. Determine the number of cars that travel through the intersection in 3 minutes.



9. **MONEY** A top film actor made 16 films, which grossed over  $\$5.09 \times 10^9$ . On average, how much did each film make? Write in scientific notation. Round to the nearest tenth.
10. **DISPLAYS** Identical boxes are stacked in the corner of a store as shown below. How many boxes are *not* visible?



11. **REMODELING** How many square feet of wallpaper are needed to cover a wall that measures 9 feet by 16 feet?
12. **STANDARDIZED TEST PRACTICE**  
Yolanda deposited \$450 in a new savings account in January, withdrew \$175 in February, and then began monthly deposits of \$75 from March through December. Which equation shows her balance  $b$  after her last deposit in December?
- (A)  $b = \$275 + 9(\$75)$   
 (B)  $b = \$275 + 10(\$75)$   
 (C)  $b = \$450 + 10(\$75)$   
 (D)  $b = \$275 - 10(\$75)$

# Volume of Rectangular Prisms

## What You'll LEARN

Find the volumes of rectangular prisms.

## NEW Vocabulary

volume  
rectangular prism

## Link to READING

**Everyday Meaning of Volume:** bulk or mass, as in shipping a large volume of merchandise

## HANDS-ON Mini Lab

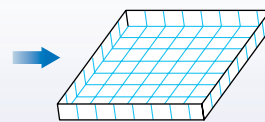
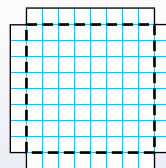
Work with a partner.

- On a piece of grid paper, cut out a square that is 10 centimeters on each side.
- Cut a 1-centimeter square from each corner. Fold the paper and tape the corners together to make a box.

## Materials

- centimeter grid paper
- scissors
- tape
- centimeter cubes

1. What is the area of the base, or bottom, of the box? What is the height of the box?

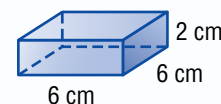


2. How many centimeter cubes fit in the box?

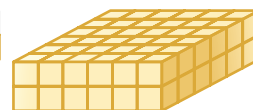
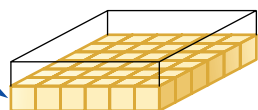
3. What do you notice about the product of the base area and the height of the box?

The **volume** of a solid is the measure of space occupied by it. It is measured in cubic units such as cubic centimeters ( $\text{cm}^3$ ) or cubic inches ( $\text{in}^3$ ).

The volume of the figure at the right can be shown using cubes.



The bottom layer, or base, has  $6 \cdot 6$  or 36 cubes.



There are two layers.

It takes  $36 \cdot 2$  or 72 cubes to fill the box. So, the volume of the box is 72 cubic centimeters.

A **rectangular prism** is a solid figure that has two parallel and congruent sides, or bases, that are rectangles.

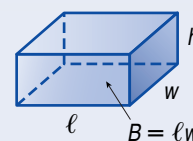
## Noteables

### Key Concept: Volume of a Rectangular Prism

#### Words

The volume  $V$  of a rectangular prism is the area of the base  $B$  times the height  $h$ . It is also the product of the length  $\ell$ , the width  $w$ , and the height  $h$ .

#### Model



#### Symbols

$$V = Bh \text{ or } V = \ell wh$$

## READING in the Content Area

For strategies in reading this lesson, visit [msmath2.net/reading](http://msmath2.net/reading).

## EXAMPLES

## Find the Volumes of Prisms

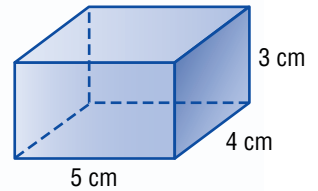
- 1 Find the volume of the rectangular prism.

$$V = \ell wh \quad \text{Volume of a rectangular prism}$$

$$V = 5 \cdot 4 \cdot 3 \quad \text{Replace } \ell \text{ with 5, } w \text{ with 4, and } h \text{ with 3.}$$

$$V = 60 \quad \text{Multiply.}$$

The volume is 60 cubic centimeters.



- 2 **CEREAL** Find the volume of the cereal box.

$$\text{Estimate } V \approx 8 \cdot 3 \cdot 10 \text{ or } 240$$

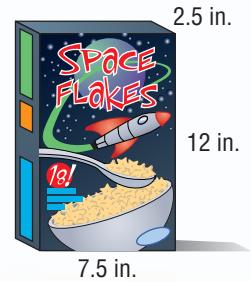
$$V = \ell wh \quad \text{Volume of a rectangular prism}$$

$$V = 7.5 \cdot 2.5 \cdot 12 \quad \text{Replace } \ell \text{ with 7.5, } w \text{ with 2.5, and } h \text{ with 12.}$$

$$V = 225 \quad \text{Multiply.}$$

The volume is 225 cubic inches.

Compare to the estimate.



## STUDY TIP

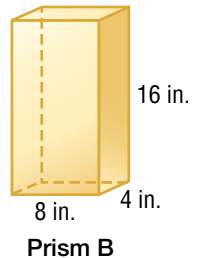
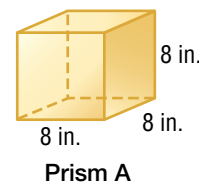
**Estimation** It is helpful to first estimate the volume of a rectangular prism to determine whether your answer is reasonable.

## Skill and Concept Check

- Write the abbreviation for cubic yards using an exponent.
- OPEN ENDED** Draw and label a rectangular prism that has a volume between 400 and 600 cubic centimeters. State the volume of the prism.
- FIND THE ERROR** Cassandra and Ling are comparing the volumes of the two prisms at the right. Who is correct? Explain.

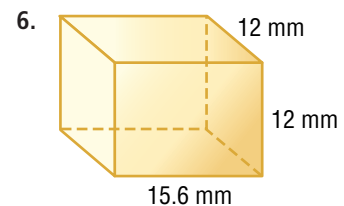
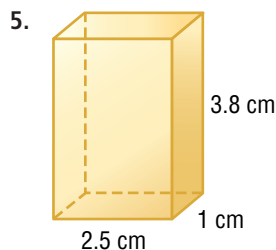
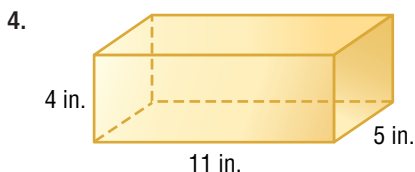
Cassandra  
Prism B has a greater volume because it is twice as tall as Prism A.

Ling  
Both prisms have the same volume.



## GUIDED PRACTICE

Find the volume of each rectangular prism.



7. **STORAGE** A cabinet measures 20 inches by 30 inches by 60 inches. What is its volume?



## Practice and Applications

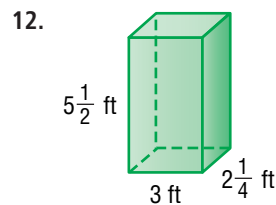
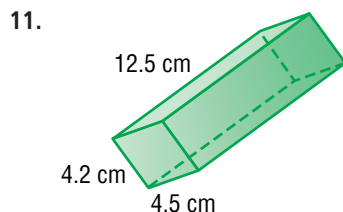
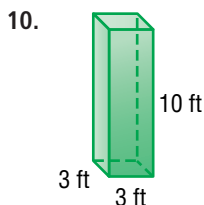
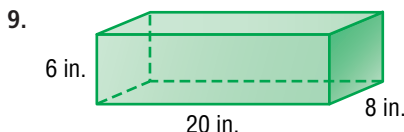
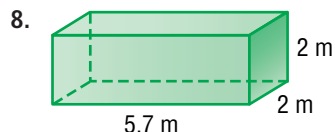
### HOMWORK HELP

For Exercises 8–15 See Examples

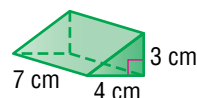
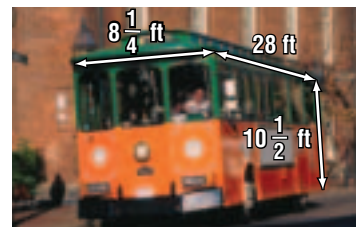
1, 2

**Extra Practice**  
See pages 593, 607.

Find the volume of each rectangular prism. Round to the nearest tenth if necessary.



13. A cube has 3-centimeter edges. Find its volume.
14. The base of a rectangular prism has an area of 19.4 square meters. Find the height if the volume is 306.52 cubic meters.
15. **TRANSPORTATION** Find the approximate volume of the California trolley car shown at the right.
16. **MEASUREMENT** How many cubic inches are in a cubic foot?
17. **MEASUREMENT** How many cubic centimeters are in a cubic meter?
18. **CRITICAL THINKING** A *triangular prism* is a prism that has bases that are triangles. Use  $V = Bh$  to find the volume of the triangular prism at the right.



## Spiral Review with Standardized Test Practice

19. **MULTIPLE CHOICE** An office is 20 feet long, 15 feet wide, and 12 feet high. It costs about 9¢ per year to air condition one cubic foot of space. On average, how much does it cost to air condition this office for one month?
- (A) \$324      (B) \$300      (C) \$27      (D) \$2.25
20. **SHORT RESPONSE** A landscaper wants to cover a 40-foot-by-12-foot rectangular area with small stones. If she uses 120 cubic feet of stones, how deep will they be?
21. Draw a top, a side, and a front view of the solid at the right.  
(Lesson 12-1)
22. Draw an area model so that the probability of a randomly dropped counter falling in a triangular region is 25%. (Lesson 11-8)



### GETTING READY FOR THE NEXT LESSON

**PREREQUISITE SKILL** Estimate. (Page 558)

23.  $3.14 \cdot 6$

24.  $5 \cdot 2.7^2$

25.  $9.1 \cdot 8.3$

26.  $3.1 \cdot 1.75^2 \cdot 2$



**What You'll LEARN**

Use a spreadsheet to investigate the volumes of similar solids.

**Similar Solids**

A computer spreadsheet can help you calculate the sizes and volumes of similar rectangular prisms. You can enlarge or reduce dimensions of the prisms and the spreadsheet will automatically calculate the new volumes.

**ACTIVITY**

Mrs. Sanchez owns a box-making factory. She wishes to produce a set of similar boxes so that they will nest inside each other when assembled. She will need to show her customers the dimensions and volume of each type of box she plans to sell. This information can be placed into a spreadsheet.



Set up the spreadsheet like the one shown below.

Similar Boxes					
	A	B	C	D	E
1	Dimensions (in.)				
2	Box	L	W	H	Volume (in <sup>3</sup> )
3	small	2.5	2.5	2.5	
4	medium	5	5	5	
5	large	7.5	7.5	7.5	
6					

**EXERCISES**

1. State the spreadsheet commands you used to find the volumes of the three boxes. Then find the volumes to the nearest tenth.
2. Expand the spreadsheet to calculate the volumes of the boxes when each of the dimensions is doubled. What happens to the volume of the small box when all of the dimensions of the box are doubled?
3. What happens to the volume of the small box when all of the dimensions of the box are tripled?
4. Suppose another box has a volume 216 times greater than the small box. What are the dimensions of this box? Use the spreadsheet to check your answer.
5. Extend the pattern in the volume column. Then express the pattern in exponential form.

# 12-3

## Volume of Cylinders

### What You'll LEARN

Find the volumes of cylinders.

### NEW Vocabulary

cylinder

### HANDS-ON Mini Lab

*Work with a partner.*

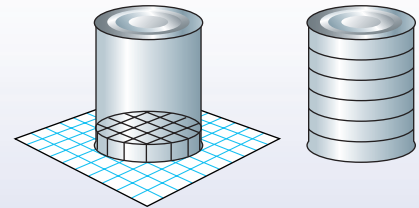
Set a soup can on a piece of grid paper and trace around the base, as shown at the right.



### Materials

- soup can
- centimeter grid paper
- scissors
- tape

1. Estimate the number of centimeter cubes that would fit at the bottom of the can. Include parts of cubes.
2. How many layers would it take to fill the cylinder?
3. **Make a conjecture** about how you could find the volume of the soup can.



A **cylinder** is a solid figure that has two congruent, parallel circles as its bases. As with prisms, the area of the base tells the number of cubic units in one layer. The height tells how many layers there are.

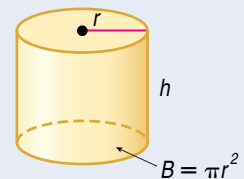
### Noteables

### Key Concept: Volume of a Cylinder

**Words** The volume  $V$  of a cylinder with radius  $r$  is the area of the base  $B$  times the height  $h$ .

**Symbols**  $V = Bh$  or  $V = \pi r^2 h$ ,  
where  $B = \pi r^2$

**Model**



### EXAMPLE

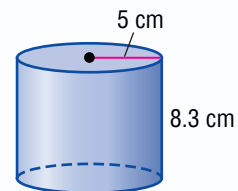
### Find the Volume of a Cylinder

- 1** Find the volume of the cylinder.  
Round to the nearest tenth.

$$V = \pi r^2 h \quad \text{Volume of a cylinder}$$

$$V = \pi(5)^2(8.3) \quad \text{Replace } r \text{ with } 5 \text{ and } h \text{ with } 8.3.$$

$$\pi \times 5 \times^2 \times 8.3 \text{ ENTER } 651.8804756$$



The volume is about 651.9 cubic centimeters. Check by using estimation.

**EXAMPLE**

**Find the Volume of a Real-Life Object**

- 1** **MULTIPLE-CHOICE TEST ITEM** Find the volume of a cylinder-shaped paint can that has a diameter of 4 inches and a height of 5 inches.  
 (A) 20.0 in<sup>3</sup>    (B) 62.8 in<sup>3</sup>    (C) 125.7 in<sup>3</sup>    (D) 251.3 in<sup>3</sup>

**Test-Taking Tip**

**Reading**

The radius of the can is  $4 \div 2$  or 2 inches. Be sure to substitute that value into the formula. If you substitute 4 for  $r$  into the formula, you would get the incorrect answer D.

**Read the Test Item**

To find the volume, use the formula  $V = \pi r^2 h$ .

**Solve the Test Item**

$V = \pi r^2 h$       Volume of a cylinder

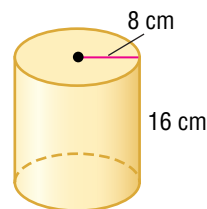
$V = \pi(2)^2(5)$     Replace  $r$  with 2 and  $h$  with 5.

$V \approx 62.8$       Simplify.

The answer is B.

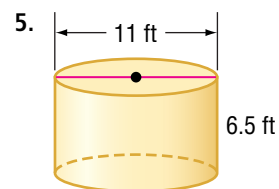
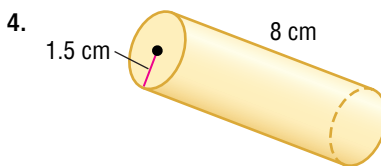
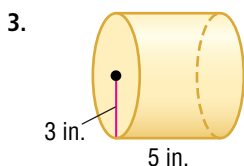
**Skill and Concept Check**

- OPEN ENDED** Draw and label a cylinder that has a larger radius, but less volume than the one shown at the right.
- Writing Math** Explain how the formula for the volume of a cylinder is similar to the formula for the volume of a rectangular prism.



**GUIDED PRACTICE**

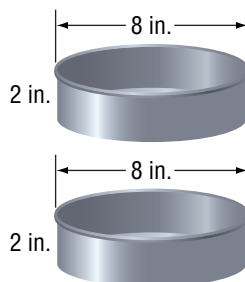
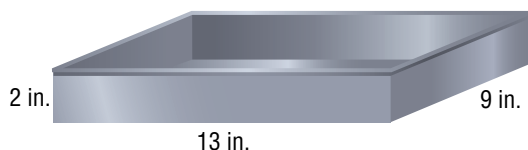
Find the volume of each cylinder. Round to the nearest tenth.



6. radius = 3 in.  
height =  $5\frac{1}{2}$  in.

7. diameter = 12.4 m  
height = 2 m

8. **BAKING** Which will hold more cake batter, the rectangular pan, or the two round pans? Explain.



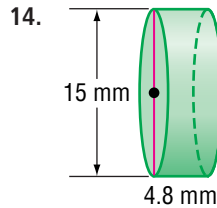
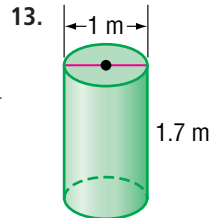
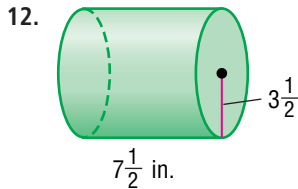
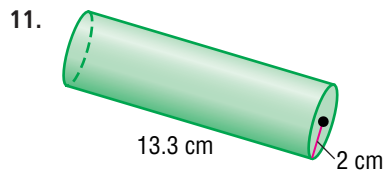
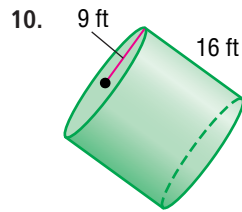
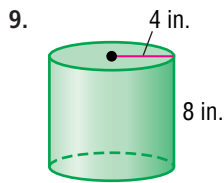
# Practice and Applications

## HOMWORK HELP

For Exercises	See Examples
9–18	1
19–22, 26–27	2

**Extra Practice**  
See pages 594, 607.

Find the volume of each cylinder. Round to the nearest tenth.



15. diameter = 21 mm  
height = 8 mm

16. diameter = 4.5 m  
height = 6.5 m

17. radius = 6 ft  
height =  $5\frac{1}{3}$  ft

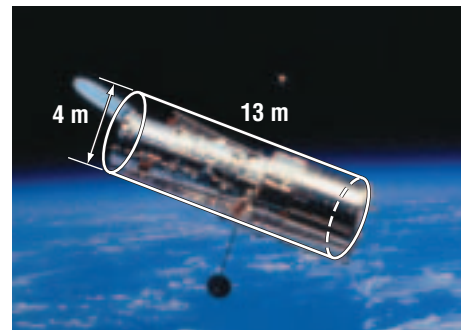
18. radius = 12 ft  
height = 11 yd

19. a soup can with a radius of 4 centimeters and a height of 12 centimeters

20. a hockey puck with a diameter of 3 inches and a height of 1 inch

21. a can of potato chips with a radius of  $1\frac{1}{2}$  inches and a height of 8 inches

22. **SPACE SCIENCE** The Hubble Space Telescope is cylinder-shaped, approximately the size of a school bus, as shown at the right. What is the volume to the nearest tenth?

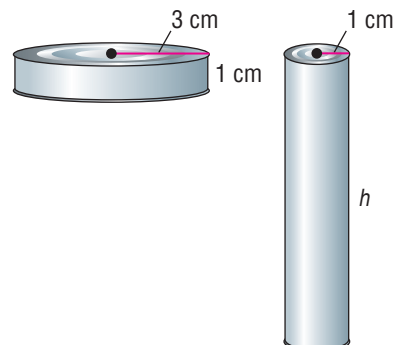


23. **WRITE A PROBLEM** Write a problem in which you find the volume of a cylinder.

24. **NUMBER SENSE** What is the ratio of the volume of a cylinder to the volume of a cylinder having twice the height but the same radius?

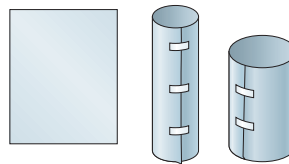
25. **NUMBER SENSE** What is the ratio of the volume of a cylinder to the volume of a cylinder having the same height but twice the radius?

26. **CONTAINERS** The two cans at the right have the same volume. Find  $h$ .



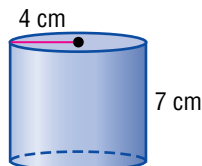
27. **MEASUREMENT** Firewood is usually sold by a unit of measure called a *cord*. A cord is a stack of wood that is 8 feet long, 4 feet wide, and 4 feet high. Suppose a tree has a diameter of 2 feet. Find the height of the tree trunk that would produce about 1 cord of firewood.

28. **CRITICAL THINKING** Two equal-sized sheets of paper are rolled along the length and along the width, as shown at the right. Which cylinder do you think has the greater volume? Explain.

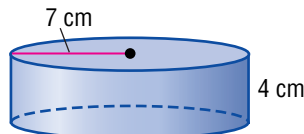


## Spiral Review with Standardized Test Practice

29. **MULTIPLE CHOICE** Which statement is true about the volumes of cylinders 1 and 2 shown below?



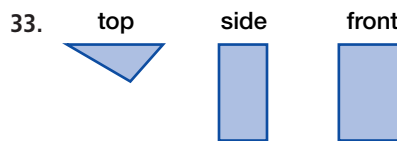
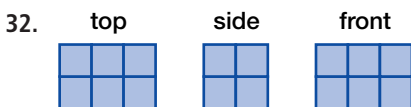
Cylinder 1



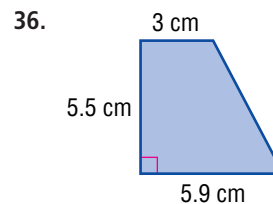
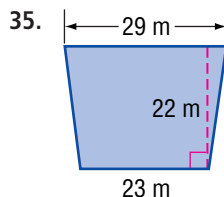
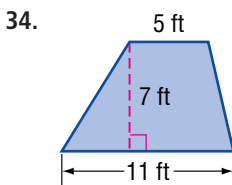
Cylinder 2

- (A) The volume of cylinder 1 is greater.  
 (B) The volume of cylinder 2 is greater.  
 (C) The volumes are equal.  
 (D) cannot tell from the diagrams
30. **MULTIPLE CHOICE** A cylinder-shaped popcorn tin has a height of 1.5 feet and a diameter of 10 inches. Find the volume to the nearest cubic inch. Use 3.14 for  $\pi$ .
- (F) 118 in<sup>3</sup>      (G) 565 in<sup>3</sup>      (H) 1,413 in<sup>3</sup>      (I) 5,652 in<sup>3</sup>
31. Find the volume of a rectangular prism with a length of 6 meters, a width of 4.9 meters, and a height of 5.2 meters. (Lesson 12-2)

Draw each solid using the top, side, and front views shown. Use isometric dot paper. (Lesson 12-1)



Find the area of each trapezoid. Round to the nearest tenth if necessary. (Lesson 11-5)



### GETTING READY FOR THE NEXT LESSON

**PREREQUISITE SKILL** Simplify. (Lesson 1-3)

37.  $3 \cdot 5 \cdot 8 + 2 \cdot 9 \cdot 3$

38.  $7.6 \cdot 11 \cdot 2 + 2 \cdot 7.6 \cdot 3$

39.  $2 \cdot 7 \cdot 1.5 + 2 \cdot 4 \cdot 1.5 + 1.5 \cdot 7$

40.  $2 \cdot 2\frac{1}{4} \cdot 6 + 2 \cdot 9 \cdot 6 + 2 \cdot 2\frac{1}{4} \cdot 9$

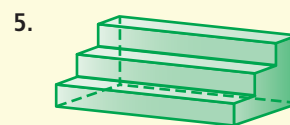
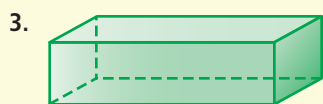


## Vocabulary and Concepts

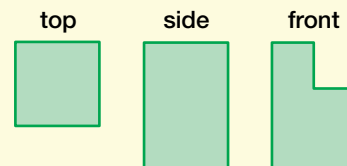
1. Define *volume*. (Lesson 12-2)
2. State, in words, the formula for the volume of a cylinder. (Lesson 12-3)

## Skills and Applications

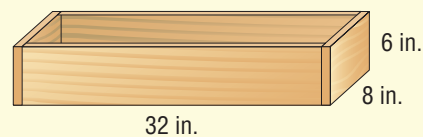
Draw a top, a side, and a front view of each solid. (Lesson 12-1)



6. Draw the solid using the top, side, and front views shown at the right. Use isometric dot paper. (Lesson 12-1)



7. **GIFTS** A jewelry box measures 7 centimeters by 12 centimeters by 14 centimeters. What is its volume? (Lesson 12-2)
8. **TRUCKS** How tall is the trailer of a truck if it is 9 meters long, 7.2 meters wide and has a volume of 226.8 cubic meters? (Lesson 12-2)
9. **GARDENING** Jocelyn is buying potting soil to fill the window box. If one bag of potting soil contains 576 cubic inches, how many bags should she buy? (Lesson 12-2)



Find the volume of each cylinder. Round to the nearest tenth. (Lesson 12-3)

10. diameter = 25 ft      11. radius = 2.2 cm      12. radius =  $1\frac{1}{2}$  in.  
 height = 24 ft      height = 5 cm      height =  $4\frac{1}{2}$  in.

## Standardized Test Practice

13. **GRID IN** A brick is 4 inches wide, 8 inches long, and 3 inches tall. What is the volume in cubic inches of a stack of 25 bricks? (Lesson 12-2)
14. **SHORT RESPONSE** A circular pond has a radius of 12 meters and it is 2.5 meters deep. What is its volume? Use 3.14 for  $\pi$ . (Lesson 12-3)

# The Game Zone

A Place To Practice Your Math Skills

**Math Skill**  
3-Dimensional Figures



## Shape-Tac-Toe

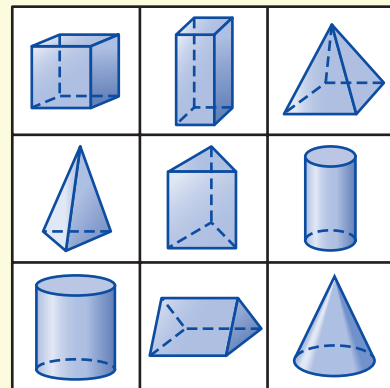
### ● GET READY!

**Players:** two

**Materials:** 1 index card, 2 large cubes, 10 two-color counters

### ● GET SET!

One player draws a game board on the index card like the one shown. The second player labels the six faces of each cube with the following words.

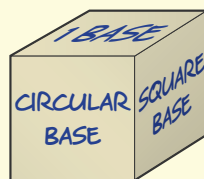


#### First Cube

prism  
pyramid  
cylinder  
cone  
flat surfaces  
curved surface

#### Second Cube

circular base  
triangular base  
square base  
2 parallel bases  
1 base  
congruent faces

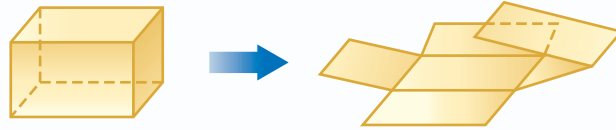


### ● GO!

- The first player rolls both cubes and places a counter on any one shape that matches the conditions on the cubes. If it is impossible to cover a shape, the player loses his or her turn.
- Players alternate turns.
- **Who Wins?** The first player to cover three shapes in a row is the winner.

## Nets and Surface Area

Suppose you cut a cardboard box along its edges, open it up, and lay it flat.



The result is a two-dimensional figure called a **net**. Nets can help you see the regions or faces that make up the surface of the figure.

### What You'll LEARN

Use nets to find the surface area of rectangular prisms.

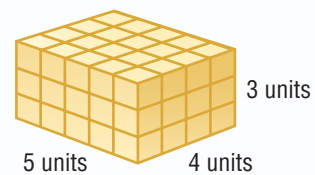
### Materials

- rectangular dot paper
- scissors

### ACTIVITY

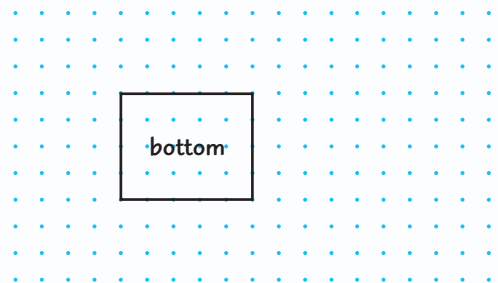
Work with a partner.

Make a net of the rectangular prism shown at the right.



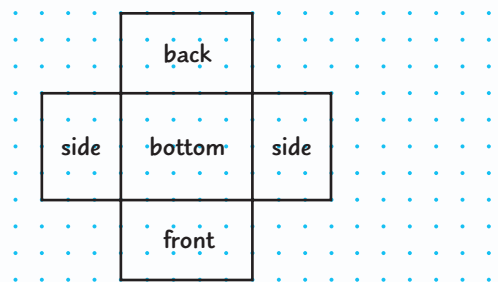
#### STEP 1

Begin by drawing the base of the prism. On the dot paper, draw a rectangle that is 5 units long and 4 units wide.



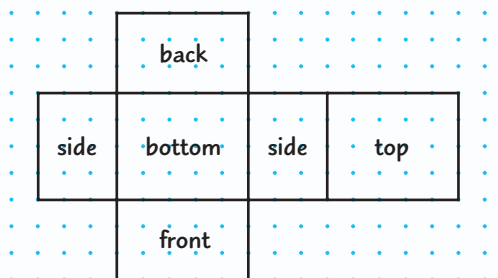
#### STEP 2

Visualize unfolding the prism along its edges. Draw the rectangles that represent the front, back, and sides of the prism.



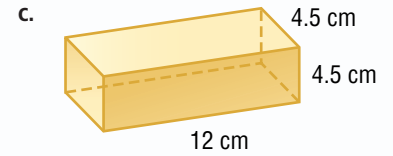
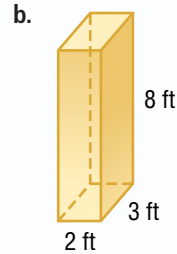
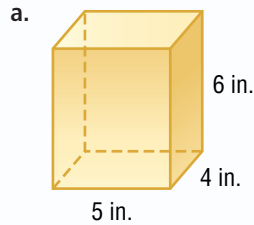
#### STEP 3

Finally, draw the top of the prism. This is only one of several possible nets that you could draw.



## Your Turn

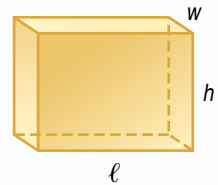
Draw a net for each figure. Find the area of the net. Then cut out the net, fold it, and tape it together to form a three-dimensional figure.



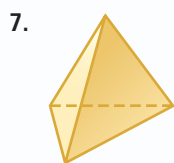
## Writing Math

Work in groups of three.

- The net shown on page 530 is made of rectangles. How many rectangles are in the net?
- Explain how you can find the total area of the rectangles.
- The *surface area* of a prism is the total area of its net. Write an equation that shows how to find the surface area of the prism at the right using the length  $\ell$ , width  $w$ , and height  $h$ .
- Find the surface areas of cubes whose edges are 1 unit, 2 units, and 3 units and graph the ordered pairs (side length, surface area) on a coordinate plane. Describe the graph.
- Describe** what happens to the surface area of a cube as its dimensions are doubled? tripled?
- Describe** how you would find the surface area of a square-based pyramid.



Draw a net for each figure.



tetrahedron



square-based pyramid

- Explain** how the formula for the surface area of a tetrahedron differs from the formula for the surface area of a square-based pyramid.

# 12-4

## Surface Area of Rectangular Prisms

### What You'll LEARN

Find the surface areas of rectangular prisms.

### NEW Vocabulary

surface area

### HANDS-ON Mini Lab

#### Materials

- 8 centimeter cubes



Work with a partner.

- Use the cubes to build a rectangular prism with a length of 8 centimeters.
  - Count the number of squares on each face. The sum is the *surface area*.
1. Record the dimensions, volume, and surface area in a table.
  2. Build two more prisms using all of the cubes. For each, record the dimensions, volume, and surface area.
  3. Describe the rectangular prisms with the greatest and least surface areas.

The sum of the areas of all of the surfaces, or faces, of a three-dimensional figure is the **surface area**.

### EXAMPLE Use a Net to Find Surface Area

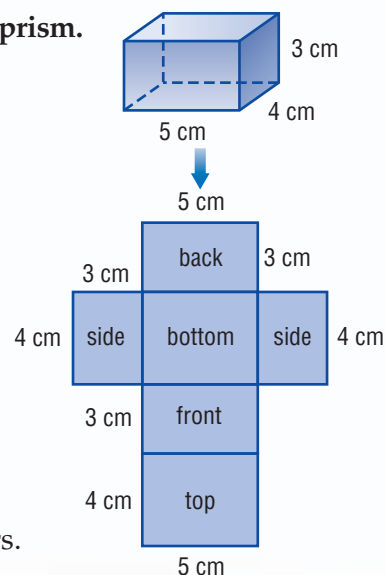
Find the surface area of the rectangular prism.

You can use a net of the rectangular prism to find its surface area. There are three pairs of congruent faces.

- top and bottom
- front and back
- two sides

Faces	Area
top and bottom	$2(5 \cdot 4) = 40$
front and back	$2(5 \cdot 3) = 30$
two sides	$2(3 \cdot 4) = 24$
Sum of the areas	$40 + 30 + 24 = 94$

The surface area is 94 square centimeters.

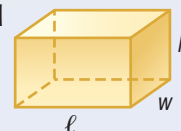


### Noteables

#### Key Concept: Surface Area of a Rectangular Prism

**Words** The surface area  $S$  of a rectangular prism with length  $\ell$ , width  $w$ , and height  $h$  is the sum of the areas of the faces.

**Model**



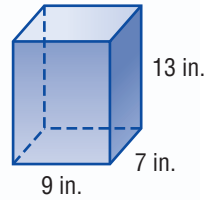
**Symbols**  $S = 2\ell w + 2\ell h + 2wh$

**EXAMPLE****Use a Formula to Find Surface Area**

- 1 Find the surface area of the rectangular prism.

Replace  $\ell$  with 9,  $w$  with 7, and  $h$  with 13.

$$\begin{aligned} \text{surface area} &= 2\ell w + 2\ell h + 2wh \\ &= 2 \cdot 9 \cdot 7 + 2 \cdot 9 \cdot 13 + 2 \cdot 7 \cdot 13 \\ &= 126 + 234 + 182 \quad \text{Multiply first. Then add.} \\ &= 542 \end{aligned}$$



The surface area of the prism is 542 square inches.

**REAL-LIFE MATH**

**GIFTS** Gifts have been wrapped since the invention of paper in 105 A.D. in China.

Source: www.hallmark.com

**EXAMPLE****Use Surface Area to Solve a Problem**

- 1 **GIFTS** Mario is wrapping a package 8 inches long, 2 inches wide, and 11 inches high. He bought a roll of wrapping paper that is 1 foot wide and 2 feet long. Did he buy enough to wrap the package?

**Step 1** Find the surface area of the package.

Replace  $\ell$  with 8,  $w$  with 2, and  $h$  with 11.

$$\begin{aligned} \text{surface area} &= 2 \cdot 8 \cdot 2 + 2 \cdot 8 \cdot 11 + 2 \cdot 2 \cdot 11 \\ &= 252 \text{ square inches} \end{aligned}$$

**Step 2** Find the area of the wrapping paper.



$$\text{area} = 12 \text{ in.} \cdot 24 \text{ in.} \text{ or } 288 \text{ in}^2$$

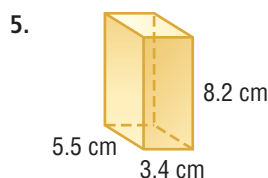
Since  $288 > 252$ , Mario bought enough wrapping paper.

**Skill and Concept Check**

1. Explain how to find the surface area of a rectangular prism.
2. **OPEN ENDED** Draw a rectangular prism and its net with the dimensions labeled on both figures. Then find the surface area.
3. **Writing Math** Explain why surface area is measured in square units even though the figure is three-dimensional.

**GUIDED PRACTICE**

Find the surface area of each rectangular prism. Round to the nearest tenth if necessary.

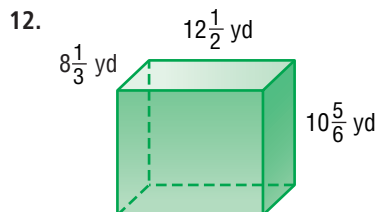
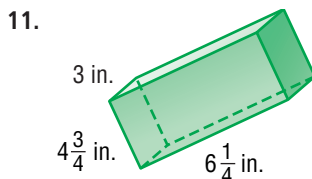
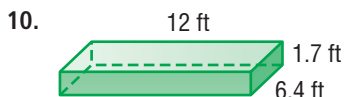
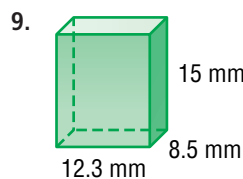
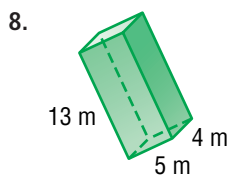
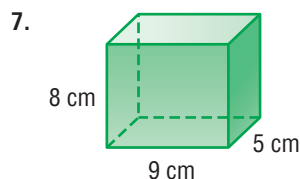


6. Find the surface area of a rectangular prism that has a length of 7 inches, a width of 11 inches, and a height of 9 inches.



## Practice and Applications

Find the surface area of each rectangular prism. Round to the nearest tenth if necessary.



13. A cube has a surface area of 42 square meters. What is the area of one face?
14. Find the surface area of a rectangular prism that has a length of 5 feet, a width of 3 feet, and a height of 9 inches.

**BOXES** For Exercises 15 and 16, use the information below.

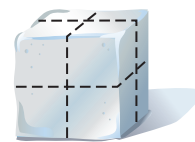
The largest corrugated cardboard box ever constructed was in Helmond, Netherlands. It measured 22.9 feet long, 8.5 feet high, and 7.87 feet wide.

15. Find the volume of the cardboard box to the nearest tenth.
16. Find the surface area of the cardboard box to the nearest tenth.
17. **ALGEBRA** Write a formula for the surface area of a cube in which each side measures  $x$  units.

**PHYSICAL SCIENCE** For Exercises 18–20, use the information below.

Granulated sugar dissolves faster in water than a sugar cube.

18. Suppose the length of each edge of a sugar cube is 1 centimeter. Find the surface area of the cube.
19. Imagine cutting the cube once in half horizontally and twice vertically. Find the total surface area of the eight cubes.
20. Make a conjecture as to why granulated sugar dissolves faster than a sugar cube.



**CEREAL** For the Exercises 21 and 22, use the following information.

Suppose you are designing a trial size cereal box that holds 100 cubic centimeters of cereal.

21. **MULTI STEP** Find the whole number dimensions of the box that would use the least amount of cardboard.
22. **MULTI STEP** If cardboard costs \$0.05 per 100 square centimeters, how much would it cost to make 100 boxes?

### HOMEWORK HELP

For Exercises	See Examples
7–14	1, 2
15–16, 18–22	3

**Extra Practice**  
See pages 594, 607.

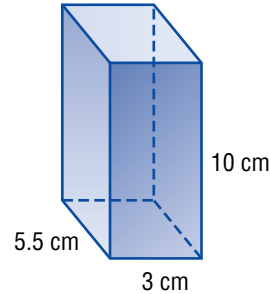


23. **WRITE A PROBLEM** Write about a real-life situation that involves finding the surface area of a rectangular prism. Then solve the problem.
24. **CRITICAL THINKING** A model is made by placing a cube with 12-centimeter sides on top of another cube with 15-centimeter sides. Find the surface area. (*Hint*: Do not include the area where the smaller cube covers the larger cube.)

## Spiral Review with Standardized Test Practice

25. **MULTIPLE CHOICE** Find the surface area of the rectangular prism to the nearest tenth.

- (A) 101.5 cm<sup>2</sup>                      (B) 165.0 cm<sup>2</sup>  
 (C) 198.6 cm<sup>2</sup>                      (D) 203.0 cm<sup>2</sup>

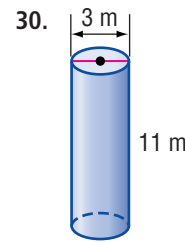
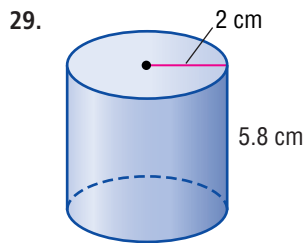
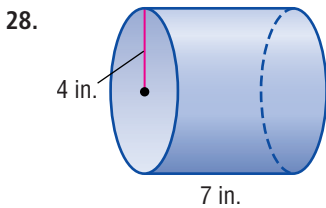


26. **MULTIPLE CHOICE** Find the amount of glass used for an aquarium that is 2.5 feet long, 1.6 feet wide, and 2 feet tall. (*Hint*: The top of the aquarium is open.)

- (F) 19.4 ft<sup>2</sup>                      (G) 20.4 ft<sup>2</sup>                      (H) 21.2 ft<sup>2</sup>                      (I) 24.4 ft<sup>2</sup>

27. **GRID IN** The surface area of a cube is 294 square millimeters. What is the length of one side in millimeters?

Find the volume of each cylinder. Round to the nearest tenth. (Lesson 12-3)

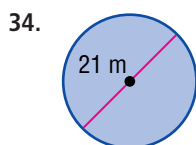
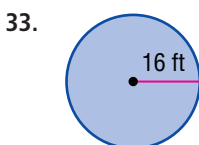


31. A rectangular prism is 14 inches long, 4.5 inches wide, and 1 inch high. Find its volume. (Lesson 12-2)

32. **CARPENTRY** The deck on a house is  $25\frac{3}{4}$  feet long and  $12\frac{1}{2}$  feet wide. The longer side of the deck is against the house. How many feet of wood does Jack need to buy to build a railing around the deck? (Lesson 6-6)

### GETTING READY FOR THE NEXT LESSON

**PREREQUISITE SKILL** Find the area of each circle. Round to the nearest tenth. (Lesson 11-6)



35. diameter = 13.6 yd    36. radius = 23 km



**What You'll LEARN**

Investigate changes in volume and surface area.

**Materials**

- isometric dot paper

**Changes in Volume and Surface Area**

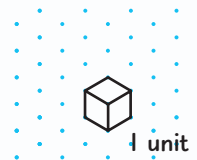
Suppose you have a model of a rectangular prism and you are asked to create a similar model whose dimensions are twice as large. In this lab, you will investigate how changing the dimensions of a three-dimensional figure affects the surface area and volume.

**ACTIVITY**

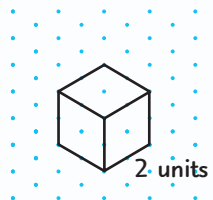
Work with a partner.

**STEP 1**

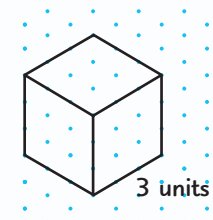
Draw a cube on dot paper that measures 1 unit on each side. Calculate the volume and the surface area of the cube. Then record the data in a table like the one shown below.

**STEP 2**

Double the side lengths of the cube. Calculate the volume and the surface area of this cube. Record the data in your table.

**STEP 3**

Triple the side lengths of the original cube. Now each side measures 3 units long. Calculate the volume and the surface area of the cube and record the data.

**STEP 4**

For each cube, write a ratio comparing the side length and the volume. Then write a ratio comparing the side length and the surface area. The first one is done for you.

Side Length (units)	Volume (units <sup>3</sup> )	Surface Area (units <sup>2</sup> )	Ratio of Side Length to Volume	Ratio of Side Length to Surface Area
1	$1^3 = 1$	$6(1^2) = 6$	1:1	1:6
2				
3				
4				
5				
s				

**Your Turn**

- a. Complete the table above.

**ACTIVITY***Work with a partner.*

- STEP 1** Draw a cube on dot paper that measures 8 units on each side. Calculate the volume and the surface area of the cube. Record the data in a table like the one shown below.
- STEP 2** Halve the side lengths of the cube in Step 1. Calculate the volume and the surface area of this cube and record the data.
- STEP 3** Halve the side lengths of the cube in Step 2. Calculate the volume and the surface area of the cube and record the data.
- STEP 4** For each cube, write a ratio comparing the side length and the volume and a ratio comparing the side length and the surface area. The first one is done for you.

Side Length (units)	Volume (units <sup>3</sup> )	Surface Area (units <sup>2</sup> )	Ratio of Side Length to Volume	Ratio of Side Length to Surface Area
8	$8^3 = 512$	$6(8^2) = 384$	8:512	8:384
4				
2				
s				

**Your Turn**

- b. Complete the table above.

**Writing Math***Work with a partner.*

- Write a formula for the volume  $V$  of a cube with side length  $s$ .
- Write a formula for the surface area  $A$  of a cube with side length  $s$ .

**Complete each sentence.**

- If the side length of a cube is doubled, the volume is ? times greater.
- If the side length of a cube is doubled, the surface area is ? times greater.
- If the side length of a cube is tripled, the volume increases by ? times and the surface area increases by ? times.
- If the side length of a cube decreases by  $\frac{1}{2}$ , the surface area decreases by ?.

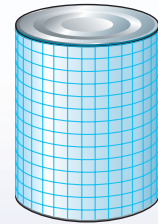
# 12-5

## Surface Area of Cylinders

### HANDS-ON Mini Lab

#### Materials

- soup can
- grid paper
- scissors



#### What You'll LEARN

Find the surface areas of cylinders.

#### REVIEW Vocabulary

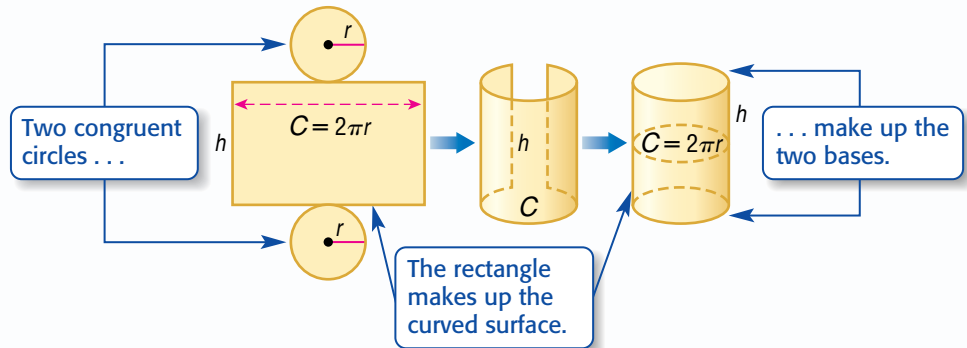
**circumference:** the distance around a circle  
(Lesson 6-9)

Work with a partner.

- Trace the top and bottom of the can on grid paper. Then cut out the shapes.
- Cut a long rectangle from the grid paper. The width of the rectangle should be the same as the height of the can. Wrap the rectangle around the side of the can. Cut off the excess paper so that the edges just meet.

1. Make a net of the cylinder.
2. Name the shapes in the net.
3. How is the length of the rectangle related to the circles?
4. Explain how to find the surface area of the cylinder.

The diagram below shows how you can put two circles and a rectangle together to make a cylinder.



In the diagram above, the length of the rectangle is the same as the circumference of the circle. Also, the width of the rectangle is the same as the height of the cylinder.

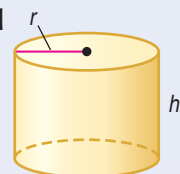
$$\underbrace{\text{The surface area of a cylinder}}_S \quad \text{equals} \quad \underbrace{\text{the area of two bases}}_{2(\pi r^2)} \quad \text{plus} \quad \underbrace{\text{the area of the curved surface.}}_{(2\pi r)h}$$

#### Noteables

#### Key Concept: Surface Area of a Cylinder

**Words** The surface area  $S$  of a cylinder with height  $h$  and radius  $r$  is the sum of the areas of the circular bases and the area of the curved surface.

**Model**



**Symbols**  $S = 2\pi r^2 + 2\pi rh$

## REAL-LIFE CAREERS

### How Does a Package Designer Use Math?

A package designer must know the size of the package before creating a design. This involves calculating the surface areas of the package.



#### Research

For information about a career as a package designer, visit: [msmath2.net/careers](http://msmath2.net/careers)

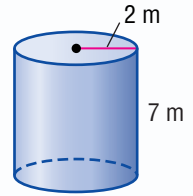


### EXAMPLE

## Find the Surface Area of a Cylinder

- 1 Find the surface area of the cylinder. Round to the nearest tenth.

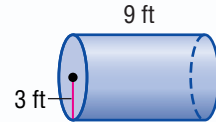
$$\begin{aligned} S &= 2\pi r^2 + 2\pi rh && \text{Surface area of a cylinder} \\ &= 2\pi(2)^2 + 2\pi(2)(7) && \text{Replace } r \text{ with 2 and } h \text{ with 7.} \\ &\approx 113.1 && \text{Simplify.} \end{aligned}$$



The surface area is about 113.1 square meters.

### Your Turn

- a. Find the surface area of the cylinder. Round to the nearest tenth.



### EXAMPLE

## Use Surface Area to Solve a Problem

- 1 **DESIGN** A can of soup is 5 inches high, and its base has a diameter of 4 inches. How much paper is needed to make the label on the can?

Since only the curved side of the can has a label, you do not need to include the areas of the top and bottom of the can.

$$\begin{aligned} S &= 2\pi rh && \text{Curved surface of a cylinder} \\ &= 2\pi(2)(5) && \text{Replace } r \text{ with 2 and } h \text{ with 5.} \\ &\approx 62.8 && \text{Simplify.} \end{aligned}$$

So, about 62.8 square inches of paper is needed to make the label.

## Skill and Concept Check

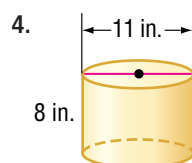
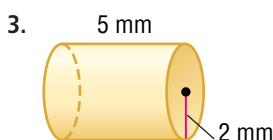
- OPEN ENDED** Find the surface area of a cylinder found in your home.
- FIND THE ERROR** Matthew and Latonya have each drawn a cylinder with the dimensions shown below and insist that theirs has more surface area. Who is correct? Explain.

Matthew  
 $r = 6 \text{ cm}, h = 3 \text{ cm}$

Latonya  
 $r = 3 \text{ cm}, h = 6 \text{ cm}$

### GUIDED PRACTICE

Find the surface area of each cylinder. Round to the nearest tenth.



5. The radius of a cylinder is 8 meters, and its height is 5 meters. Find the surface area to the nearest tenth.

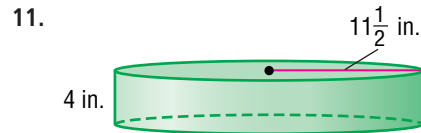
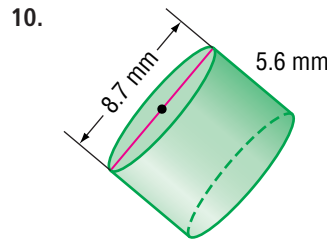
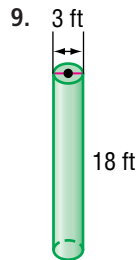
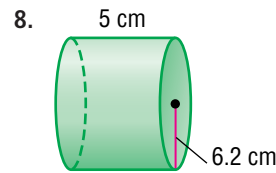
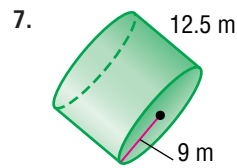
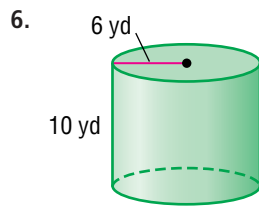


[msmath2.net/extra\\_examples](http://msmath2.net/extra_examples)



## Practice and Applications

Find the surface area of each cylinder. Round to the nearest tenth.



### HOMWORK HELP

For Exercises	See Examples
6–11	1
12–19	2

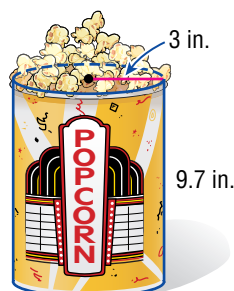
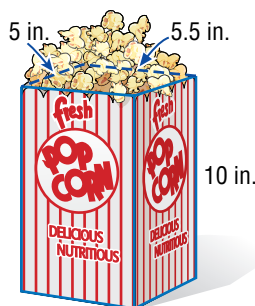
Extra Practice  
See pages 595, 607.

- Find the area of the label on a can of tuna with a radius of 5.1 centimeters and a height of 2.9 centimeters.
- The height of a water tank is 10 meters and has a diameter of 10 meters. What is the surface area of the tank?
- An underground oil tank has a surface area of 2,915 square feet and a radius of 6 feet. What is the height of the oil tank?

**BAKING** For Exercises 15 and 16, use the following information.

A *smash cake* is a smaller version of a birthday cake given to a child on his or her first birthday. Mrs. Jones baked a smash cake 3 inches high and 10 inches in diameter for her daughter when she turned 1 year old.

- Mrs. Jones covers the top and sides of the cake with frosting. Find the area that the frosting covers to the nearest tenth.
- Suppose Mrs. Jones first cuts the cake in two layers, frosts the top and sides of the bottom layer, puts the top layer on, and then frosts the rest of the cake. How much of the cake is covered with frosting now?
- MOVIES** The Student Council at Southwest Middle School is planning to sell popcorn in one of two open-top containers below at its “Movie Night.” The cost depends on the amount of cardboard used to make each container. Which container should Student Council buy? Use volume and surface area measurements to explain your choice.

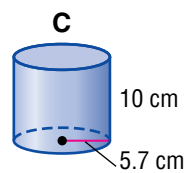
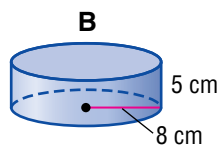
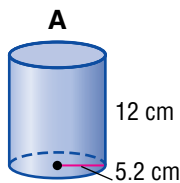


18. **CANDY** The largest piece of candy ever created was shaped like a cylinder 4.49 meters long and had a circumference of 1.14 meters. What was the surface area of the candy to the nearest tenth?
19. **MODELING** Suppose you took an  $8\frac{1}{2}$ - $\times$ -11-inch piece of paper and made two cylinders; one using the  $8\frac{1}{2}$ -inch side as the height (cylinder A) and the other using the 11-inch side as the height (cylinder B). Without calculating, which cylinder has the greater surface area? Explain.
20. **CRITICAL THINKING** If you double the height of a cylinder, will its surface area also double? Explain your reasoning.
21. **EXTENDING THE LESSON** In Lesson 12-1, you saw another solid with a curved surface, called a *cone*, as shown at the right. The curved surface is difficult to draw accurately in two dimensions since it is not a polygon. Make a cone with a piece of paper. Then draw the net for the cone using it as a pattern.



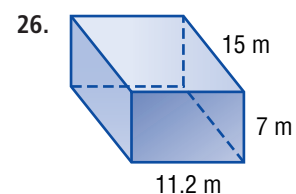
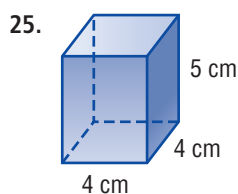
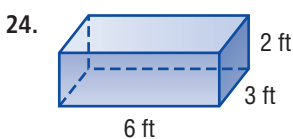
## Spiral Review with Standardized Test Practice

22. **MULTIPLE CHOICE** Gilberto completely covers a cylindrical can with construction paper. The can has a height of 15 inches and a radius of 4 inches. About how much construction paper does he use?
- (A) 477.5 in<sup>2</sup>    (B) 375.2 in<sup>2</sup>    (C) 104.5 in<sup>2</sup>    (D) 60.0 in<sup>2</sup>
23. **MULTIPLE CHOICE** The three metal containers below each hold about 1 liter of liquid. Which container has the greatest surface area?



- (F) A    (G) B    (H) C    (I) none of them

Find the surface area of each rectangular prism. (Lesson 12-4)



27. Find the volume of a cylinder having a radius of 4 inches and a height of 6.5 inches. (Lesson 12-3)

### GETTING READY FOR THE NEXT LESSON

**PREREQUISITE SKILL** Find the value of each expression. Round to the nearest tenth. (Pages 559, 560)

28.  $13.2 + 15.378$

29.  $23.7 - 9.691$

30.  $23 \cdot 7.1$

31.  $1.6(8.5)0.4$



**What You'll LEARN**

Determine and apply significant digits in a real-life context.

**NEW Vocabulary**

precision  
precision unit  
significant digits

**WHEN** am I ever going to use this?

**WORLD RECORDS** In 2000, Joyce Samuels blew a bubble gum bubble that had a diameter of 27.94 centimeters.

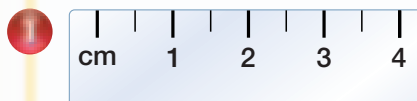
1. What is the smallest unit of measure?
2. Explain whether a ruler whose smallest increment is 0.5 centimeter could have been used to measure the bubble.



The **precision** or *exactness* of a measurement depends on the unit of measure. The **precision unit** is the smallest unit on a measuring tool. As the units get smaller, the measurement gets “more precise.”

**EXAMPLES Identify Precision Units**

Identify the precision unit of each measuring tool.



The smallest unit is a half centimeter. So, the precision unit is 0.5 centimeter.



The smallest unit is an eighth of an inch. So, the precision unit is  $\frac{1}{8}$  inch.

**Your Turn**

- a. Identify the precision unit of the scale at the right.



All measurements are approximate. You could estimate the measure on the scale below as 8 grams. A more precise method is to use significant digits. **Significant digits** include all of the digits of a measurement that you know for sure, plus one estimated digit.



estimated digit

8.3 g

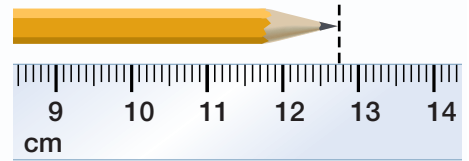
2 significant digits

digit known for certain

**EXAMPLE****Apply Significant Digits**

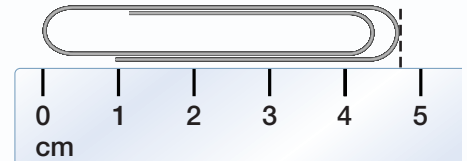
1. State the measure of the pencil using significant digits.

The precision unit is 0.1 centimeter. You know for certain that the length is between 12.7 and 12.8 centimeters. One estimate is 12.75 centimeters.



2. **Your Turn**

- b. State the measure of the paper clip using significant digits.

**Skill and Concept Check**

- Writing Math** Choose the most precise unit of measurement: *inches*, *feet*, or *yards*. Explain.
- OPEN ENDED** Describe some items or situations in which an approximate measure is sufficient. Then describe some items or situations in which exactness in measurement is important and necessary.
- Which One Doesn't Belong?** Identify the measurement that does not have the same number of significant digits as the other three. Explain your reasoning.

8.6 mi

14.5 ft

3.22 cm

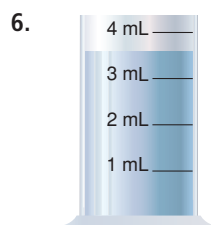
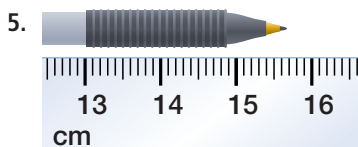
90.7 yd

**GUIDED PRACTICE**

4. Identify the precision unit of the ruler at the right.



State each measure using significant digits.



7. **OLYMPICS** Tom Dolan broke the Olympic Swimming World Record in 2000 for the 400-meter race. His time was 4 minutes, 11.76 seconds. Describe the precision of this measure.



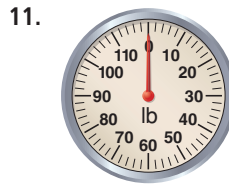
# Practice and Applications

**HOMEWORK HELP**

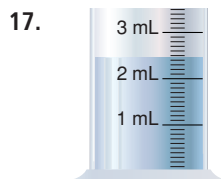
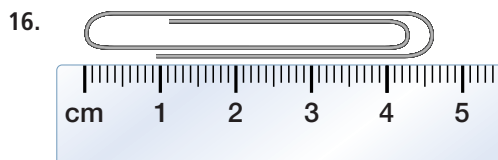
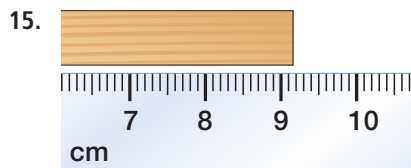
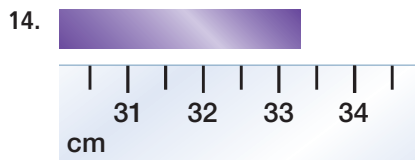
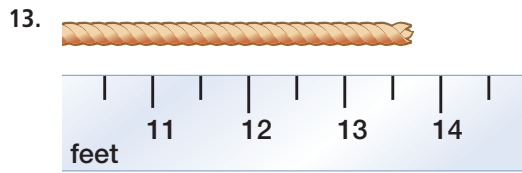
For Exercises	See Examples
8–11, 19–20	1, 2
12–18	3

**Extra Practice**  
See pages 595, 607.

Identify the precision unit of each measuring tool.



State each measure using significant digits.



**TRIATHLONS** For Exercises 18–20, use the graphic at the right.

- How many significant digits are in the measurement describing the length of the cycling portion of the triathlon?
- Which is a more precise measure: the number of miles cycled in a triathlon, or the number of miles run? Explain.
- The running portion of the women's triathlon is equivalent to 5 kilometers. Which unit is more precise: a mile or a kilometer? Explain.
- RESEARCH** The silica tiles on space shuttles are cut accurately to 10,000th of an inch. Use the Internet or another source to investigate the technology that is used to make such precise measurements.

**USA TODAY**

USA TODAY Snapshots®

**More women attempt triathlon challenge**

More than 2,000 women will swim a half-mile, cycle 12.4 miles and run 3.1 miles in the nation's third-largest triathlon – Sunday's Danskin Women's Triathlon – in Austin, Texas. Since the first Danskin race in 1990, the number of women competing in triathlons in the USA has grown significantly:

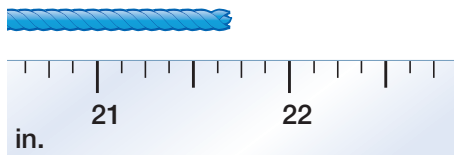
1990	26,000
1995	39,100
2000	72,000

Source: USA Triathlon

By Ellen J. Horrow and Marcy E. Mullins, USA TODAY

22. If you use a ruler with a precision unit of 1 millimeter, will your measurements *sometimes*, *always*, or *never* be exact? Explain.
23. **WRITE A PROBLEM** Write a measurement problem in which you use significant digits.

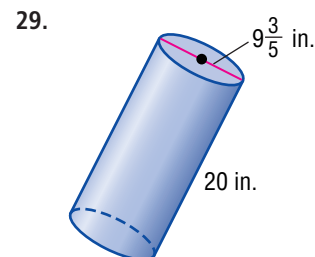
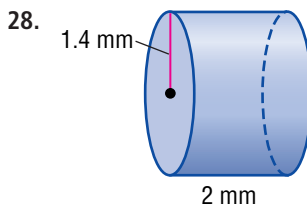
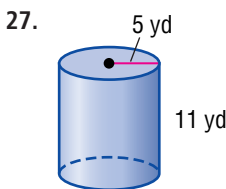
24. **CRITICAL THINKING** Choose the most precise measures to complete the sentence: The yarn at the right is between  $\underline{\quad?}$  and  $\underline{\quad?}$  inches long.



## Spiral Review with Standardized Test Practice

25. **MULTIPLE CHOICE** Choose the measuring tool that you would use to estimate the volume of a shoebox.  
 A scale       B ruler       C protractor       D measuring cup
26. **MULTIPLE CHOICE** Choose the best precision unit for estimating the length of a gymnasium.  
 F 0.1 cm       G 0.5 in.       H 0.1 mi       I 0.5 ft

Find the surface area of each cylinder. Round to the nearest tenth. (Lesson 12-5)



30. Find the surface area of a rectangular prism that has a length of 19 centimeters, a width of 8.8 centimeters, and a height of 13 centimeters. (Lesson 12-4)

**ALGEBRA** Solve each equation. Check your solution. (Lesson 6-5)

31.  $\frac{n}{15} = 3$

32.  $-26 = \frac{x}{2.4}$

33.  $\frac{3}{8}y = 9$

### INTERDISCIPLINARY PROJECT

#### It's All Greek to Me

**Math and History** It's time to complete your project. Use the information and data you have gathered about Pythagoras to prepare a Web page or poster. Be sure to include the three-dimensional solid you created with your project.



[msmath2.net/webquest](http://msmath2.net/webquest)



[msmath2.net/self\\_check\\_quiz](http://msmath2.net/self_check_quiz)



## Vocabulary and Concept Check

cylinder (p. 524)  
precision (p. 542)

precision unit (p. 542)  
rectangular prism (p. 520)

significant digits (p. 542)  
solid (p. 514)

surface area (p. 532)  
volume (p. 520)

Choose the correct term or number to complete each sentence.

1. A (rectangular prism, rectangle) is a solid figure that has three sets of parallel congruent sides.
2. The (volume, surface area) of a solid figure is the measure of the space occupied by it.
3. Volume is measured in (square, cubic) units.
4. The volume of a rectangular prism is found by (adding, multiplying) the length, the width, and the height.
5. A (cylinder, prism) is a solid that has two congruent, parallel circles as its bases.
6. Surface area is the sum of the (areas, volumes) of all of the outside surfaces of a three-dimensional figure.
7. The (smaller, larger) the precision unit, the more precise is the measurement.
8. Significant digits include all of the digits of a measurement that you know for sure, plus (one, two) estimated digit(s).

## Lesson-by-Lesson Exercises and Examples

## 12-1 Drawing Three-Dimensional Figures (pp. 514–517)

Draw a top, a side, and a front view of each solid.

9.



10.

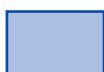


11. Draw a solid using the top, side, and front views shown. Use isometric dot paper.

top



side

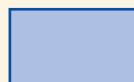


front



**Example 1** Draw the solid using the top, side, and front views shown.

top



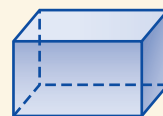
side



front



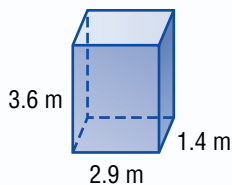
The side view is a square. The top and front views are rectangles.



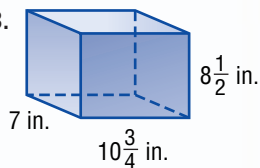
## 12-2 Volume of Rectangular Prisms (pp. 520–522)

Find the volume of each rectangular prism. Round to the nearest tenth.

12.

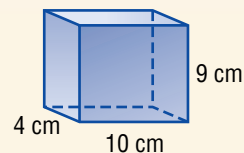


13.



14. **POOLS** A swimming pool 25 yards long has 8 lanes that are each 3 yards wide. The water is 6 feet deep. Find the volume of water in the pool.

**Example 2** Find the volume of the rectangular prism.



$$V = \ell wh \quad \text{Volume of a rectangular prism}$$

$$V = (10)(4)(9) \quad \ell = 10, w = 4, \text{ and } h = 9.$$

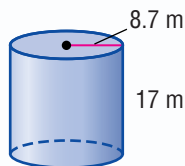
$$V = 360 \quad \text{Multiply.}$$

The volume is 360 cubic centimeters.

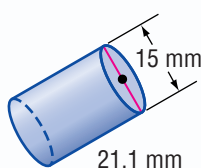
## 12-3 Volume of Cylinders (pp. 524–527)

Find the volume of each cylinder. Round to the nearest tenth.

15.

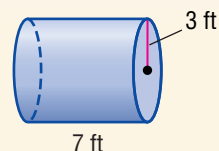


16.



17. **POTTERY** In art class, Arturo made a vase in the shape of a cylinder. The diameter is 5 inches, and the height is 10 inches. Find the maximum volume of water the vase can hold.

**Example 3** Find the volume of the cylinder.



$$V = \pi r^2 h \quad \text{Volume of a cylinder}$$

$$= \pi (3)^2 (7) \quad \text{Replace } r \text{ with } 3 \text{ and } h \text{ with } 7.$$

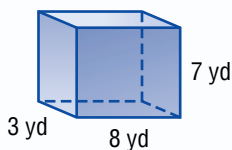
$$\approx 197.9 \quad \text{Multiply.}$$

The volume is about 197.9 cubic feet.

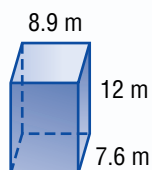
## 12-4 Surface Area of Rectangular Prisms (pp. 532–535)

Find the surface area of each rectangular prism. Round to the nearest tenth if necessary.

18.

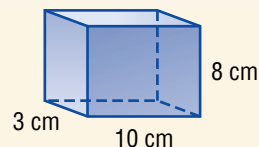


19.



20. **PETS** A plastic pet carrier box is 2.5 feet long, 1 foot high, and 1.25 feet wide. How much plastic is used to make this carrier?

**Example 4** Find the surface area of the rectangular prism.



$$\text{surface area} = 2\ell w + 2\ell h + 2wh$$

$$= 2(10)(3) + 2(10)(8) + 2(3)(8)$$

$$= 268$$

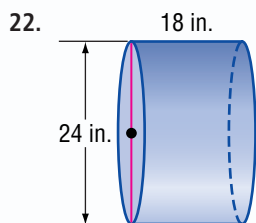
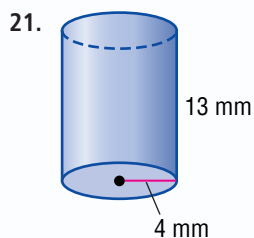
The surface area is 268 square centimeters.

**Mixed Problem Solving**

For mixed problem-solving practice, see page 607.

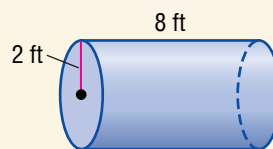
**12-5 Surface Area of Cylinders** (pp. 538–541)

Find the surface area of each cylinder. Round to the nearest tenth.



23. **DESIGN** A can of chicken noodle soup is  $5\frac{1}{2}$  inches high, and its base has a radius of 2 inches. How much paper is needed to make the label on the can?

**Example 5** Find the surface area of the cylinder.

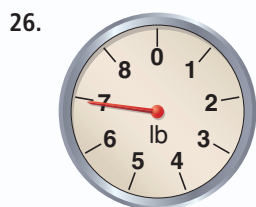
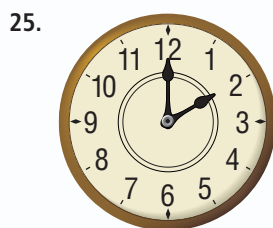
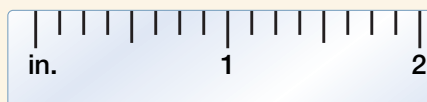


$$\begin{aligned} \text{surface area} &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(2)^2 + 2\pi(2)(8) \\ &\approx 125.7 \text{ ft}^2 \end{aligned}$$

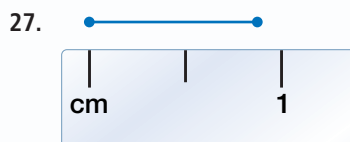
The surface area is about 125.7 square feet.

**12-6 Measurement: Precision** (pp. 542–545)

Identify the precision unit of each measuring tool.



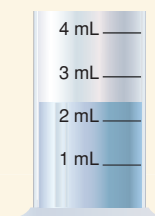
State each measure using significant digits.



**Example 6** Identify the precision unit of the measuring tool.

The smallest unit is one eighth of an inch. So, the precision unit is  $\frac{1}{8}$  inch.

**Example 7** State the measure of the graduated cylinder using significant digits.



The precision unit is 1 milliliter. You know for certain that the volume is between 2 and 3 milliliters. One estimate is 2.4 milliliters.



## Vocabulary and Concepts

1. Define *surface area*.
2. Explain what a precision unit is.

## Skills and Applications

Draw a top, a side, and a front view of each solid.

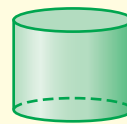
3.



4.

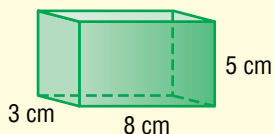


5.



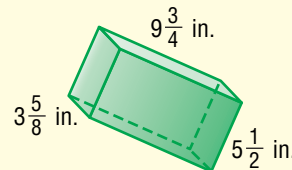
Find the volume and surface area of each rectangular prism and cylinder. Round to the nearest tenth if necessary.

6.



7. rectangular prism;  
length = 19.6 m  
width = 14 m  
height = 26.1 m

8.



9. cylinder;  
radius = 6 ft  
height = 12 ft

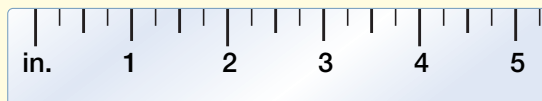
10.  6 in.  $\frac{3}{4}$  in.

11. cylinder;  
diameter = 11.5 mm  
height = 20.7 mm

12. **DRINKING STRAWS** A drinking straw has a radius of  $\frac{1}{8}$  inch and a height of  $7\frac{3}{4}$  inches. How much liquid is in a straw that is half full?
13. **PACKAGING** What is the least amount of paper needed to wrap a box that is 9 inches by 18 inches by 4 inches?

Identify the precision unit of each measuring tool.

14.



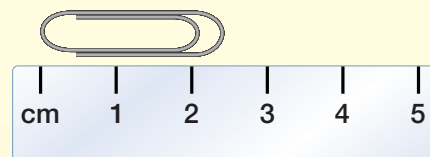
15.



## Standardized Test Practice

16. **MULTIPLE CHOICE** Find the best estimate of the measure of the paper clip using significant digits.

- (A) 2 cm                      (B) 2.25 cm  
(C) 2.4 cm                    (D) 2.75 cm



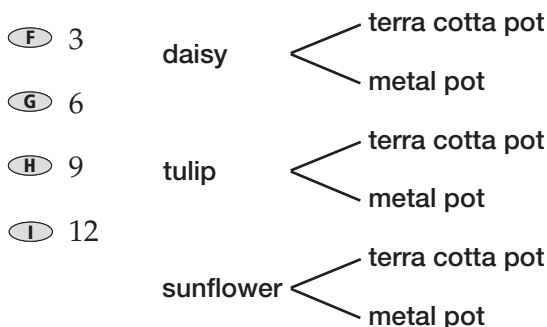
**PART 1** Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

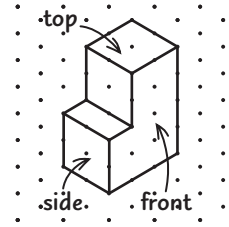
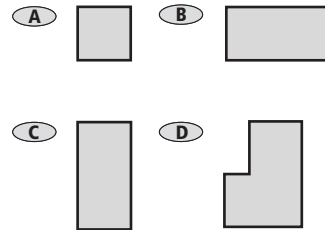
1. Which city in the table is the closest to sea level? (Lesson 3-2)

City	Altitude Compared to Sea Level (ft)
Ocean City	-25
Redmond	1,376
Reston	24
West Orange	-13

- (A) West Orange      (B) Reston  
 (C) Ocean City      (D) Redmond
2. What is the value of  $w$  in the equation  $6w + 12 = 30$ ? (Lesson 4-4)
- (F) 3      (G) 12      (H) 18      (I) 30
3. Each inch of Tyrell's train set represents  $4\frac{1}{2}$  feet of an actual train system. If the engine is 6 inches long, how long is an actual engine? (Lesson 7-4)
- (A)  $1\frac{1}{3}$  ft      (B) 27 in.  
 (C)  $10\frac{1}{2}$  ft      (D) 27 ft
4. The tree diagram shows the choices of flowers and pots sold at Ivitz Gardens. How many different flower-pot choices are possible? (Lesson 9-2)



5. Which is the side view of the solid shown at the right? (Lesson 12-1)

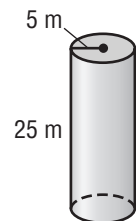


6. Gillian has a window box the shape of a rectangular prism. It is 36 inches wide and 6 inches long. How much soil should she buy in order to fill the flower box with a 4-inch-deep layer of soil? (Lesson 12-2)

- (F)  $46 \text{ in}^3$       (G)  $576 \text{ in}^3$   
 (H)  $864 \text{ in}^3$       (I)  $1,296 \text{ in}^3$

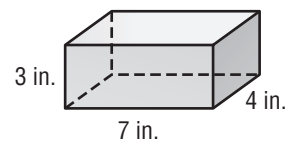
7. Which is the volume of the cylinder? (Lesson 12-3)

- (A)  $94.2 \text{ m}^3$   
 (B)  $1,963.5 \text{ m}^3$   
 (C)  $3,125.0 \text{ m}^3$   
 (D)  $9,812.5 \text{ m}^3$



8. What is the surface area of the rectangular prism? (Lesson 12-4)

- (F)  $84 \text{ in}^2$   
 (G)  $122 \text{ in}^2$   
 (H)  $141 \text{ in}^2$   
 (I)  $588 \text{ in}^2$



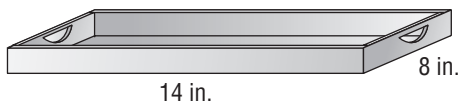
**TEST-TAKING TIP**

**Questions 6-8** Most standardized tests include any necessary formulas in the test booklet. It helps to be familiar with formulas such as the volume or surface area of a cylinder, but use any formulas that are given to you.

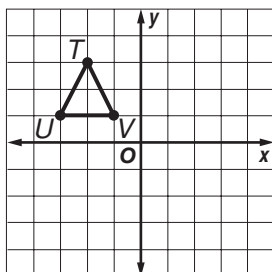
**PART 2 Short Response/Grid In**

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

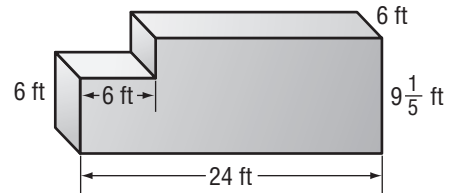
9. To find the perimeter of the rectangular tray shown below, Oi Ying wrote the equation  $2 \times 8 + 2 \times 14$ . What operation should Oi Ying do first? (Lesson 1-3)



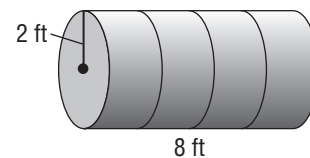
10. Marcos worked at a car wash during the summer. He worked with other employees to wipe the cars dry. Typically, Marcos wiped down 75% of the cars each day. What are two other ways that 75% can be expressed? (Lesson 5-6)
11. Find  $5\frac{1}{2} \div \frac{3}{2}$ . (Lesson 6-6)
12. At the Gilmour family reunion, 28 of the 70 family members ate potato salad. What percent of the Gilmours ate potato salad at the reunion? (Lesson 7-5)
13. At a sale, Shay finds a \$125 coat marked down to \$87.50. What percent of decrease is this? (Lesson 8-4)
14. Alisa wants a bicycle that usually costs \$186. During a sale, the bicycle sells for 20% less than that. What is the price of the bicycle when it is on sale? (Lesson 8-5)
15. Triangle  $TUV$  is reflected over the  $x$ -axis. What are the new coordinates of point  $T$ ? (Lesson 10-9)



16. Find the total volume of the solid. (Lesson 12-2)



17. What is the surface area of a rectangular prism that is 14.2 centimeters long, 10.5 centimeters wide, and 3 centimeters high? (Lesson 12-4)
18. The cylindrical gasoline storage tank on Mrs. Simon's farm needs to be painted. The dimensions are shown below.

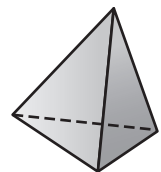


If one quart of paint covers 85 square feet, how many quarts does Mrs. Simon need for this job? (Lesson 12-5)

**PART 3 Extended Response**

Record your answers on a sheet of paper. Show your work.

19. Draw a top view, a front view, and a side view of the *tetrahedron* shown at the right. (Lesson 12-1)



20. Determine whether the given precision unit is appropriate to use in estimating the length of each object. Explain your reasoning. If it is not, describe an appropriate precision unit. (Lesson 12-6)
- driveway: 1 inch
  - compact disc: 1 meter
  - calculator key: 0.1 centimeter